Closed-Loop Gain Analysis of LC Quadrature VCO for the Accurate Prediction of Oscillation Frequency

Haizheng Guo, Bangli Liang, Bo Wang, Dianyong Chen, Tadeusz Kwasniewski
Department of Electronics
Carleton University
Ottawa, Canada
hguo1@doe.carleton.ca, bliang@doe.carleton.ca, bwang@doe.carleton.ca, chendianyong@gmail.com, tak @doe.carleton.ca

Abstract—An accurate feedback system analysis method is used as a tool for analyzing submicron CMOS LC-tank Quadrature VCOs. A simplified analysis for the closed-loop gain and oscillation frequency is developed, and good agreement is found between the feedback theorem and simulation results. The insight closed-loop gain is used to explore circuit design trade-offs.

Keywords—QVCO; feedback system; closed-loop gain; Nodal; Middlebrook’s Method;

I. INTRODUCTION

Multimode multiband capability becomes essential in modern telecommunications systems. A low power, low phase noise differential voltage-controlled oscillator (VCO) is not sufficient anymore. The availability of accurate quadrature signals becomes a prerequisite for the highly integrated low-cost receivers in standard CMOS [1]. As a result, the deeper insight into VCO performance and the study of quadrature generation has attracted the interest of many researchers.

One method to get quadrature outputs is to let the LC-tank VCO work at double the desired frequency [2]. The quadrature signals can be gotten at the desired frequency via frequency division. However the main drawback is that the frequency-dividing circuitry consumes a lot of power. A more attractive and popular approach to achieve quadrature signals is coupling two symmetric LC-tank VCOs to each other, as shown in the Fig. 1. The combination of a direct connection and a cross connection forces the two VCOs oscillate in quadrature. The two-core solution can furthermore provide a very high voltage swing, which eases the design of prescaler and mixer circuits somehow connected to the VCO [3].

This LC-tuned quadrature VCO (QVCO) shown in the Fig.1 is basically a feedback network. Oscillation will occur at the frequency at which loop transfer function is exactly one or the impedance of the LC-tank becomes infinity. Many recent papers analyze the oscillation frequency and closed-loop gain of QVCOs based on conventional feedback theorem. Unfortunately, this approach can give incorrect results. In this work, an accurate feedback analysis method is employed to analyze the oscillation frequency and closed-loop gain of the QVCO. In the section 2, the incorrectness of conventional feedback theorem has been discussed. Then an accurate feedback system theorem was used as a tool to analyze the closed-loop gain and the oscillation frequency of the LC QVCO. Finally, the noise performance of the QVCO was discussed based on the simulation results which rely on the previous feedback system theorem. Section 3 addresses the conclusion of this work.

II. FEEDBACK SYSTEM ANALYSIS

A. Conventional feedback system analysis method

The conventional way to analyze a feedback system is breaking the closed loop and getting open loop gain first, as shown in the familiar block diagram of Fig. 2. The arrowhead shapes imply that the signal goes only one way.

Where the feedback ratio is $K$, and the loop gain $T = AK$ can be calculated. According to the conventional analysis method, the designer’s job is to set feedback ratio $K$ and the forward gain $A$ so that the final closed-loop gain $H$ meets a specification. The closed-loop gain $H$ is given by

$$H = \frac{A}{1 + AK}$$

which can be better inferred as

$$H = \frac{1}{K} \frac{AK}{1 + AK} = \frac{T}{1 + T} = H_\infty \frac{1}{1 + \frac{1}{H_\infty}} = H_\infty D$$

(1)

(2)
where

\[ H_\infty \equiv \frac{1}{K} \equiv \text{ideal closed-loop gain} \]

\[ T \equiv AK \equiv \text{loop gain} \]

This conventional block diagram of Fig. 2 is an incomplete representation of the actual hardware feedback system because it does not account for bidirectional signal transmission in the blocks.

Simulating the closed-loop gain of a feedback system by opening the loop could bring two problems. First, the DC operation points on both sides of the breakpoints after opening are usually different. Second, the small-signal AC impedances seen on both sides are different from the ones in closed-loop case. The first problem can be solved by closing the loop again with a large inductor and then injecting the signal with a large capacitor. The second problem can be approached by adding a replica of the circuit on the other side of the opening. However, in many cases this only provides an approximation to the actual closed-loop impedance. In general, opening the loop to simulate loop gain is an inexact and error-prone method.

**B. Closed-Loop feedback system theorem**

To avoid potential problems mentioned above, Middlebrook’s Method [4] is employed in this work to find out the accurate closed-loop gain. Michael Tian also proposed loop-based and device-based algorithms for stability analysis of linear analog circuits in the frequency domain [5].

However, both Middlebrook’s Method and Tian’s algorithms can only be applied to circuits with exactly one single-ended loop. In order to extend these methods to symmetric differential circuits, we can connect two ideal baluns back-to-back with their differential-mode and common-mode ports and insert this combination into the circuit. Then we can insert the probe components into the path between the differential-mode ports in order to simulate the differential-mode loop gain or between the common-mode ports in order to simulate the loop gain of the common-mode regulation.

To understand Middlebrook’s Method, Fig. 3 displays the block diagram for a feedback amplifier. The gain may be calculated with either voltage or current gain. The current and voltage designators for the block diagram are as follows: source (is and vs), input (ii and vi), feedback (if and vf), and output (io and vo). The open loop voltage gain and the open loop current gain are defined as:

\[ A_v = \frac{v_o}{v_s} \quad A_i = \frac{i_o}{i_s} \]  

The system voltage gain and system current gain are defined as:

\[ A_v \cdot f = \frac{A_v}{1 + A_v \cdot B} \quad A_i \cdot f = \frac{A_i}{1 + A_i \cdot B} \]  

where, B is the feedback transfer ratio. The quantity A×B is the loop gain. A positive loop gain suggests that the feedback is negative. A negative loop gain means that the feedback is positive which may lead to oscillations in a VCO circuit. The loop gain is also equivalent to the following:
Now it is not necessary to break the loop to measure these gains. The loop may be opened in its feedback path and the appropriate test signal injected. Injecting a current into the signal path will split the current into its input and feedback currents. The ratio of the feedback and input current can then be measured to produce a current loop gain. The voltage gain may also be measured by using the same technique with placing a voltage source in the loop.

For the actual measurements, macros will be created for the voltage and current injections in order to measure the voltages and currents with user functions. Both the voltage loop gain and the current loop gain must be taken into account to measure the total loop gain. They are related through the following equation:

$$G_r = \frac{v_r}{v_i} \quad G_t = \frac{i_r}{i_t} \quad (5)$$

This equation may be reduced to:

$$G = \frac{G_r \cdot G_t - 1}{G_r + G_t + 2} \quad (6)$$

As is normal in parallel calculations, the lower of the current or voltage gain will be the one that dominates. The QVCO circuit to be analyzed is shown in Fig.4. The circuit is shown in its closed loop measurement configuration.

The QVCO circuit was implemented in IBM 0.13 $\mu$m technology. To maximize speed and minimize tank capacitive load, all transistors length were set to minimal length. The widths of the transistors of the cores were dimensioned for maximal amplitude and for maximal symmetry of the sine waveforms. The widths of cross-coupling transistors were firstly set to be the same with the core transistors. By injecting error voltage source and error current source separately, we got the simulation results of error voltage gain and error current gain, along with both real and imaginary part, which are shown in the Fig. 5.

In order to get the closed-loop gain of the QVCO, these data were imported into Matlab to calculate final gain by using (6). The main part of QVCO closed-loop gain is shown in the Fig.6. From the whole plot of loop gain, we can find two pairs of poles. Around the oscillation frequency there are two peaks, which are located at 9.1GHz (major peak) and 10GHz (minor peak) separately.

C. Nodal simulation of the feedback system

To verify this result, we used Nodal in Mathematica to calculate the closed-loop transfer function of the QVCO which is shown in Fig. 4, and we also obtained four poles as: $\pm1.208 \times 10^{10} \pm 2.588 \times 10^9 j$, $\pm8.714 \times 10^{10} \pm 2.133 \times 10^9 j$. The magnitude and phase of the feedback network can be seen from the Fig. 7. The oscillation frequency can be found at around 10.6GHz.

Both Spectre simulation and Nodal analysis show that the closed-loop transfer function of the LC-QVCO has four poles. The oscillation frequency simulated by Nodal analysis is higher than the one gotten from Spectre simulations. The oscillation frequency gotten from Nodal is more close to the nature frequency of the QVCO, which is $1/\sqrt{LC}$. While in the real oscillator, there is a frequency shift with respect to the nature frequency. To explain the frequency difference, let’s consider the linear model for the QVCO shown in the Fig. 8.

Fig. 8 introduces a linear model for the QVCO, where $G_M$ represents the transconductance of the negative-resistance pair, and $G_{d0}$ represents the transconductance of the cross-coupling.
pair. Referring to $V_x$, the losses in tank-$X$ are balanced by a current ($I_i$) provided by $G_{M}$, which is in phase with $V_x$. Because the tank is lossless, the current from $G_{M}$ acts on an ideal LC-parallel. This second current ($I_Q$) is thus in quadrature with $V_x$, which implies that $V_x$ and $V_y$ are phase shifted by $\pi/2$ [6].

Fig. 9 shows the phase shift of the voltage across the tank, a second of the currents entering the tank. The frequency shift could be either positive or negative depending on the inductive or capacitive characteristic of $I_Q$, which is established by the nonlinear mechanisms in the real oscillator and cannot be obtained from the linear model.

D. Noise performance analysis of the QVCO

This feedback analysis method can also benefit the noise performance analysis for the QVCO.

According to our simulation and calculation, when the widths of the cross-coupling transistors decrease, the second peak (minor one) in the plot of closed-loop gain shown in Fig.6 disappears gradually. We can therefore get pure single tone oscillation frequency. Also the phase noise, too, greatly decreases, while the phase error gets quickly larger. Thus, it is straightforward to improve the phase noise performance of the QVCO at the expense of its phase error performance.

This is the case in [3] presented by Tiebout, where a very high phase noise FoM was achieved by choosing cross-coupling width one third of the width of core-transistors. This also proved that the closed-loop gain analysis method based on the presented feedback system theorem is feasible and intuitionistic.

III. CONCLUSION

In this work, instead of using conventional feedback system analysis method, an accurate feedback theorem was used for analyzing oscillation frequency as well as closed-loop gain of LC QVCO. This approach is simple and proved valid by using both Nodal and Spectre. It is also straightforward when optimizing noise performance of the QVCO.

REFERENCES